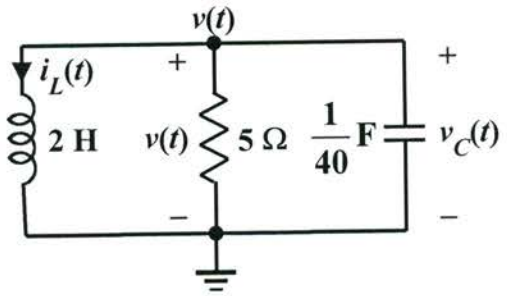
EE 2240

Problem #02

For the underdamped circuit shown, determine the current $i_L(t)$ if the initial conditions on the storage elements are $i_L(0) = 1$ A and $v_C(0) = 10$ V.



From KCL: $i_L + \frac{v}{5} + \frac{1}{40}\frac{dv}{dt} = 0$ [1]

Note that $v = 2\frac{di_L}{dt}$, so that the KCL equation may be rewritten as:

$$i_L + \frac{2}{5}\frac{di_L}{dt} + \frac{1}{20}\frac{d^2 i_L}{dt^2} = 0$$

or $$\frac{d^2 i_L}{dt^2} + 8\frac{di_L}{dt} + 20 i_L = 0$$

The corresponding characteristic equation is:

$$r^2 + 8r + 20 = 0 \quad \text{or} \quad r = -4 \pm j2$$

Therefore, $i_L(t) = e^{-4t}(K_1 \cos 2t + K_2 \sin 2t)$

From [1], $\left.\frac{di_L}{dt}\right|_{t=0} = \frac{1}{2} v(0) = 5$

Solving for K_1 and K_2, we have

$$i_L(t) = e^{-4t}\left(\cos 2t + \frac{9}{2}\sin 2t\right) \text{ A}, \quad t \geq 0$$